La mer 28: 67-77, 1990
Société franco-japonaise d'océanographie, Tokyo

Reflection of electromagnetic waves at sea surface*

Takashi ICHIYE**

Abstract: Reflection of plane electromagnetic waves (EMW) at the sea surface is treated based on Maxwell's theory. It is assumed that oil or freshwater of a uniform depth covers infinitely deep sea.

For dielectric constants and conductivity of oil or freshwater, reflectivity (R) or phase lag (PL) of reflected waves decrease or increase almost linearly with thickness of the cover layer up to several centimeters. The critical thickness decreases with increasing frequency and incident angle from the vertical and is smaller for freshwater than for oil. Thus, the thickness of oil in case of oil spills or that of freshwater caused by precipitation in the Tropical Pacific Ocean before El Niño may be determined with SAR (Synthetic Aperture Radar) or SLAR (Side Looking Airborne Radar) at frequencies around 1 GHz. If the thickness is known otherwise, the precise measurements of R and PL may estimate aging of oil or mixing of freshwater with seawater, by determining conductivity and dielectric constants of oil or freshwater. EMW of 20 GHz or higher frequencies is not effective to detect the freshwater but is useful to determine dielectric constant of oil for estimation of its aging. This is because EMW are reflected at the oil film, whose thickness has a variance of a few millimeters but at the same reflection angle and because the mean R of such reflected waves depends only on the incident angle and dielectric constant of oil.

1. Introduction

It is believed that reflection and refraction of electromagnetic waves (EMW) at the sea surface are a well-known problem solved as an exercise of the classical electromagnetic theory. However, the recent development and wide application of satellite and airborne remote sensing with microwaves of the ocean surface processes suggests that more careful check of the problem may be warranted to understand information obtained with such techniques. It is found that existing literature in both oceanography and electronic engineering lacks in addressing to this fundamental problem in depth, though it abounds in details on oceanic processes and technical treatment of hardwares and signal processing. Classical study by LIEBERMANN (1962) dismissed EMW as a tool for exploring the oceanic processes except possibly with extremely low frequency waves, because of electric conductivity of the sea water which attenuates the incident EMW amplitudes to insignificance within centimeters for microwave range, though SLATER (1942) noted that the sea water behaves between conductor and insulator for that range. Development of synthetic aperture (SAR) or side-looking airborne radar (SLAR) has stimulated microwave remote sensing of the ocean environment with a quantum leapfrog, since these devices could overcome hindrances of cloud cover inherent to the passive sensors both in IR and microwave ranges.

However, the fundamental assumption on which these techniques depend is that the sea surface reflects the incident EMW as a perfect conductor, though the surface is rugged due to gravity waves and ripples of various lengths and heights (TREBITS, 1987: ELACHI, 1988). PHILLIPS (1988) reviewed application of SAR to oceanic processes, particularly to determining ocean wave spectra and noticed many ambiguous problems in interpretation of SAR images simply as Bragg reflection of EMW at the multifaceted sea surface caused by ripples.

This paper is limited to the problem of reflection of EMW at a flat sea surface but is mainly addressed to two practical situations different from the implicit assumption of the sea surface as the perfect conductor. They are a thin layer of oil or fresh water covering

---

*Received August 29, 1989

**Department of Oceanography, Texas A & M University, College Station, TX 77843, U.S.A.

the infinitely deep layer of the sea water. The oil film case is applicable to oil spills that occurred as the disastrous but by no means unexpected event in Prince Williams Sound, Alaska in March 1989. The fresh water upper layer covers the Tropical Pacific Ocean over a wide area and its usual coverage and seasonal change are crucial to early detection of El Niño. This is because the event has been reported to start as the eastward propagating equatorial Kelvin waves from the western Equatorial Pacific Ocean (Gill, 1982). In the latter area an excessive precipitation causes sea level height anomalies in the form of fresh water accumulated at the sea surface (Ichiye and Muneyama, 1989). It is impossible to monitor the fresh water accumulation over the vast tropical oceanic area without satellite remote sensing. In both cases the reflection of EMW particularly in micro-wave range is a crucial problem to understand, in order to interpret imagery data available with the current technology and to improve the devices available now or to develop new ones.

2. Basic equations

The thichkness of oil or fresh water is denoted h (Fig. 1) and the underlying sea water is assumed infinite in depth. The z-axis is positive downward. The plane EMW is incident with an angle θ to the vertical. The y-component (out of paper) of magnetic field is expressed by

$$H_{jy}=(a_j e^{-u_j z}+b_j e^{u_j z})e^{-\iota\lambda x+\iota\omega t}, \qquad (1)$$

where j=0, 1, and 2 refer to the atmosphere (vacuum), the upper layer and the lower layer, respectively, and λ is an arbitrary positive parameter, a_j and b_j represent amplitude of incident and reflected waves, and ω is a circular frequency of EMW. When a_0 is prescribed, b_j, u_j and λ can be determined in terms of ω, θ and electromagnetic constants of three media through Maxwell's equations and boundary conditions (Stratton, 1941).

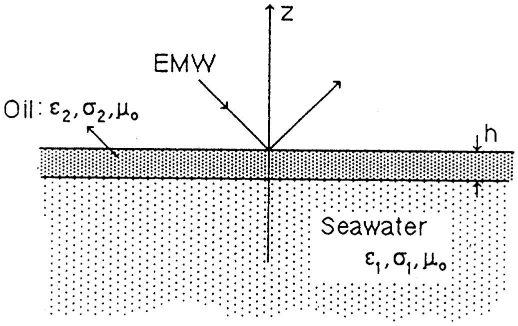

Fig. 1. Schematical figure of EMW reflected by a thin layer of oil on the sea water.

Hereafter the exponential term of equation (1) is dropped. The main problem of reflection of EMW at the sea surface is to discuss the reflectivity b_o/a_o. Wait (1962) derived this ratio for multi-layered media in terms of transmission line theory which seems to be familiar among electronic engineers. However, for three layers, the advantage of simplification on serial expressions is lost, thus here a traditional approach that is familiar with oceanographers and physicists is taken.

First, Maxwell's equation leads to

$$u_j^2=\lambda^2+\gamma_j^2, \qquad (2)$$
$$\gamma_j^2=\iota\,\sigma_j\mu_j\omega-\varepsilon_j\mu_j\omega^2, \qquad (3)$$

where σ_j, ω_j and μ_j denotes conductivity, dielectric constant and magnetic susceptivity, respectively and the real part of μ_j is always positive.

The boundary conditions are that H_jy and the x-component of electric field E_jx are continuous, whereas

$$E_{jx}=-(\sigma_j+\iota\,\omega\,\varepsilon_j)^{-1}\partial H_{jy}/\partial z. \qquad (4)$$

In terms of a_j and b_j, these conditions are expressed with

$$a_0+b_0=a_1+b_1 \qquad (5a)$$
$$K_0(-a_0+b_0)=K_1(-a_1+b_1)\,(\text{at } z=0) \qquad (5b)$$
$$a_1 e^{-u_1 h}+b_1 e^{u_1 h}=a_2 e^{-u_2 h} \qquad (6a)$$
$$K_1(-a_1 e^{-u_1 h}+b_1 e^{u_1 h})=K_2(-a_2 e^{-u_2 h}) \quad (\text{at } z=h), \qquad (6b)$$

where

$$K_j=u_j(\sigma_j+i\omega\varepsilon_j)^{-1},\ j=0,\ 1,\ 2. \qquad (7)$$

In the lower layer there is no reflected wave, thus $b_2=0$.

Equations (5) and (6) form four linear equations about a_0, a_1, a_2, and b_1. Thus the four a_j and b_j can be expressed in terms of a_0. The amplitude and phase of the reflected wave can be represented by the magnitude of complex ratio b_0/a_0 and its phase. The result of derivation from (5) and (6) is given by

$$b_0/a_0 = M/N \tag{8a}$$
$$M = (K_0 - K_2) - (K_1 - K_0 K_2 K_1^{-1}) \tanh\alpha \tag{8b}$$
$$N = (K_0 + K_2) + (K_1 + K_0 K_2 K_1^{-1}) \tanh\alpha \tag{8c}$$
$$\alpha = u_1 h. \tag{8d}$$

The argument u_2h does not appear in (8), since at $z=h$ the r.h.s. of equation (6) lacks b_2-term.

3. Electromagnetic properties of the sea water, fresh water and oil

In relations (8a)-(8d), parameters K_j and α can be expressed in terms of θ and electromagnetic constants. In such expressions realistic electromagnetic properties of the atmosphere, sea water, oil and fresh water are taken into account. First, the atmosphere is considered as insulator, thus $\sigma_0 \equiv 0$. Then oil, fresh water and sea water are all nonmagnetic. Thus $\mu_0 = \mu_1 = \mu_2$, and $\mu_0 = \mu$. Further ε_0 is considered as the value in vacuum and its suffix is dropped. Then

$$\gamma_0^2 = -\mu\,\varepsilon\,\omega^2 = -(\omega/c)^2, \tag{9}$$

where c is speed of light in vacuum. Equation (1) and Fig. 1 indicate that incident and reflected waves have a factor $e^{-i\lambda x}$ leading to

$$\lambda = (\omega/c)\sin\theta. \tag{10}$$

Substitution of (9) and (10) into (2) yields

$$u_0 = (\omega/c)\cos\theta. \tag{11}$$

This with (1) confirms that a_0 and b_0 are the amplitude of incident and reflected waves, respectively.

Substitution of (11) into (7) leads to

$$K_0 = (\mu/\varepsilon)^{\frac{1}{2}}\cos\theta = \kappa\cos\theta, \tag{12}$$

where

$$\kappa = (\mu/\varepsilon)^{\frac{1}{2}}. \tag{13}$$

The expressions of u_1, K_1, and K_2 become simplified because of values of electromagnetic constants of oil, freshwater and seawater.

When s_j and δ_j are defined by

$$s_j = \varepsilon_j/\varepsilon \qquad (j=1,\ 2) \tag{14a}$$
$$\delta_j = \sigma_j/\varepsilon_j \qquad (j=1,\ 2). \tag{14b}$$

u_j and Kj are expressed by

$$u_j = i(s_j\mu\varepsilon)^{\frac{1}{2}}\omega(1 - s_j^{-1}\sin^2\theta - i\omega^{-1}\delta_j)^{\frac{1}{2}}, \qquad (j=1,\ 2), \tag{15}$$

$$K_j = \kappa(s_j)^{\frac{1}{2}}(1 - s_j^{-1}\sin^2\theta - i\omega^{-1}\delta_j)^{\frac{1}{2}}(1 - i\delta_j)^{-1}, \qquad (j=1,\ 2), \tag{16}$$

For the fresh water and the sea water, $s_1 \approx s_2 \approx 80$. therefore the $\sin\theta$ term in (15) and (16) can be neglected against unity. Thus (15) and (16) become

$$u_j = i(s_j\mu\varepsilon)^{\frac{1}{2}}\omega(1 - i\omega^{-1}\delta_j)^{\frac{1}{2}}, \quad (j=1,\ 2) \tag{17}$$

$$K_j = \kappa(s_j)^{\frac{1}{2}}(1 - i\omega^{-1}\delta_j)^{-\frac{1}{2}}, \quad (j=1,\ 2). \tag{18}$$

Oil has dielectric constant ranging from $2.2\,\varepsilon$ to $4\,\varepsilon$ in its component hydrocarbon liquids (KAYE and LABY, 1988). The approximate expression (17) and (18) for oil are not so precise compared to exact one (15) and (16) as for the freshwater and the seawater, However, it is expected that oil spilled on the sea usually is mixed with the sea water, thus its dielectric constant may increase and here it is assumed (15)´ and (16) are applicable to oil too.

The δ_j-term in (15) to (18) represents effects of conductivity on EMW reflection and refraction by the medium. If this term is neglible against unity the medium is insulator, whereas if it is much larger than unity, the medium is conductor. This term depends also on the frequency of EMW, thus even if the medium is insulator for the high frequency, it may become conductor for the low frequency EMW. The value of δ_j is thus considered as a critical frequency, which divides the medium as insulator or conductor.

For instance, the sea water with $\sigma_2 = 4\ \Omega^{-1}\,m^{-1}$, δ_2 becomes 0.9GHz (Giga Hertz), with $\varepsilon_2 \approx 80$ (MKS unit). Therefore for GHz wave the sea water is neither conductor nor insulator. On the other hand for oil σ_1 is 2×10^{-8} to $10^{-10}\ \Omega^{-1}\,m^{-1}$ at 20℃ (LANDOLT and BÖRNSTEIN, 1960). Therefore δ_1 of oil is about 1.8×10^2 Hz and thus microwaves reflect at the oil surface as from insulator. For the fresh water the same calculation yields the critical frequency as 10^5 Hz; thus it behaves like insulator for microwaves.

The feature that the seawater is neither insulator nor conductor in microwave frequency range of GHz was recognized by SLATER (1942). However, this frequency range is widely used for both SAR and other remote sensing with satellite for oceanic processes because of requirement of high frequency range necessary for high horizontal resolution of imaging targets on the sea. Particularly for SAR and SLAR an impractically large antenna is needed to obtain resolution of orders of several tens of meters both in azimuth and range planes (ELACHI, 1988).

4. Reflectivity and phase lag of the waves reflected at the sea surface

Since the δ_2-term is near unity in equation (18) for microwave range in K_2, this is expressed by

$$K_2 = u + iv = \kappa (s_2)^{-\frac{1}{2}} (1 - i\omega^{-1}\delta_2)^{-\frac{1}{2}}, \tag{19}$$

where u and v and both positive and dependent on frequency.

The reflectivity (R) and phase lag (PL) of the reflected waves can be expressed by R_m and ϕ_m as

$$b_0/a_0 = R_m e^{-i\phi_m} = M_m/N_m, \quad (m=1,\ 2), \tag{20}$$

where m=1 and 2 represent the oil covered sea and the fresh water covered sea, respectively. With approximations that both oil and fresh water are insulator for frequency range considered (δ_1-term neglected against unity in equations (19) and (20)) and that oil's ε_1 value is taken as 4ε ($\sin\theta$ terms are neglected against unity in equations (15) and (16)), M_m and N_m become

$$M_m = (\cos\theta - pu)\cos\beta_m - q_m v\cos\theta\sin\beta_m + i\{pv\cos\beta_m + (s_m - q_m u\cos\theta)\sin\beta_m\}, \quad (m=1,\ 2) \tag{21}$$

$$N_m = (\cos\theta + pu)\cos\beta_m - q_m v\cos\theta\sin\beta_m + i\{pv\cos\beta_m + (s_m + q_m u\cos\theta)\}\sin\beta_m, \quad (m=1,\ 2). \tag{22}$$

Parameters in (21) and (22) are defined as below:

$$p = (s_2)^{-\frac{1}{2}} \approx (80)^{-\frac{1}{2}} \approx 0.1118 \tag{23a}$$

$$q_1 = (s_1/s_2)^{\frac{1}{2}} \approx 2p, \quad q_2 = 1 \tag{23b}$$

$$\beta_1 = (\omega/c)(s_1 - \sin^2\theta)^{\frac{1}{2}} h \approx (\omega/c) s_1^{\frac{1}{2}} h \approx 0.42\Gamma h \tag{23c}$$

Table 1. Part of R and PL versus β/π. Gamma (frequency in GHz), N is β/π, Mag is R and Phase is minus PL. Symbols h_r and h_p denote: where R and PL become extreme for the first time.

INDEX = 1 GAMMA = 0.8

θ(°) →	0.		15.		30.		45.		60.	
N(β/π)	Mag	Phase	Mag	Phase	Mag	Phase	Mag	Phase	Mag	Phase
0.00	0.847	0.002	0.842	0.002	0.825	0.003	0.790	0.004	0.716	0.008
0.05	0.841	-0.011	0.837	-0.010	0.820	-0.008	0.786	-0.004	0.716	0.008
0.10	0.831	-0.023	0.826	-0.023	0.809	-0.020	0.776	-0.012	0.716	0.008
0.15	0.813	-0.036	0.808	-0.035	0.792	-0.030	0.762	-0.019	0.716	0.008
0.20	0.786	-0.049	0.781	-0.048	0.767	-0.041	0.743	-0.026	0.716	0.008
0.25	0.748	-0.062	0.744	-0.059	0.733	-0.049	0.719	-0.029	0.716	0.008
0.30	0.697	-0.071	0.695	-0.067	0.690	hp -0.054	0.691	hp -0.030	0.716	0.008
0.35	0.632	hp -0.073	0.633	hp -0.068	0.640	-0.052	0.662	-0.026	0.716	0.008
0.40	0.562	-0.059	0.569	-0.054	0.592	-0.038	0.638	-0.016	0.716	0.008
0.45	0.511	-0.020	0.523	-0.017	0.559	-0.010	0.623	-0.001	0.716	0.008
0.50	hr 0.510	0.035	hr 0.522	0.032	hr 0.559	0.025	hr 0.623	0.017	0.716	0.008
0.55	0.560	0.074	0.567	0.069	0.591	0.054	0.637	0.032	0.716	0.008
0.60	0.630	0.089	0.632	0.084	0.639	0.068	0.662	0.042	0.716	0.008
0.65	0.696	0.088	0.694	0.084	0.689	0.071	0.690	0.046	0.716	0.008
0.70	0.747	0.078	0.743	0.075	0.732	0.066	0.718	0.046	0.716	0.008
0.75	0.785	0.066	0.781	0.064	0.766	0.057	0.742	0.042	0.716	0.008
0.80	0.812	0.053	0.807	0.052	0.791	0.047	0.762	0.036	0.716	0.008
0.85	0.830	0.040	0.825	0.039	0.809	0.036	0.776	0.029	0.716	0.008
0.90	0.841	0.027	0.836	0.027	0.820	0.025	0.785	0.021	0.716	0.008
0.95	0.847	0.014	0.842	0.014	0.825	0.014	0.790	0.012	0.716	0.008
1.00	0.847	0.002	0.842	0.002	0.825	0.003	0.790	0.004	0.716	0.008

$$\beta_2 \approx (\omega/c)(s_2)^{\frac{1}{2}}h \approx 1.873\Gamma h \qquad (23d)$$

$$(1-i\,0.8993\Gamma^{-1})^{-\frac{1}{2}} = u + iv, \qquad (23e)$$

where in (23c) and (23d) Γ is the frequency of EMW in GHz and h is the thickness of oil or freshwater in cm.

Equations (20) (21) and (22) indicate that R and PL are periodic about β_m with a period of π, whereas β_m are proportional to Γh as given by (23c) and (23d) for oil and fresh water, respectively. Therefore, when R and PL are given for the β_m from 0 to π, their values for different h can be computed by use of (23c) and (23d) at different frequencies.

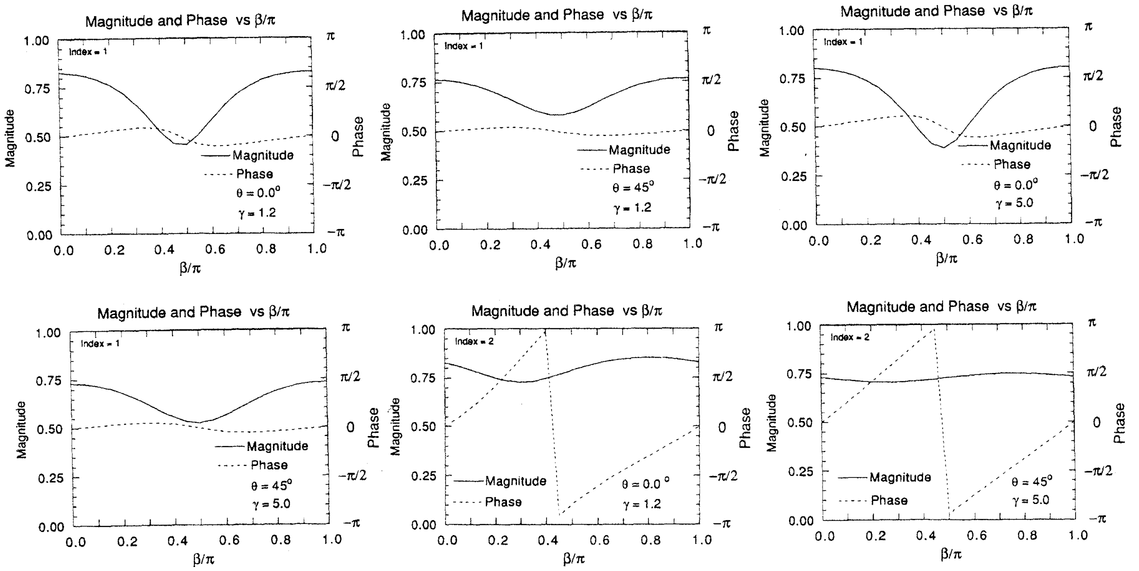

Fig. 2. Examples of R and PL versus beta for oil (A, Index 1) and fresh water (B, Index 2) at 1.2 and 4 GHz for θ = 0° and 45°.

Table 2. Part of R and PL versus h (H here) in cm. See explanation of table 1. Symbols h_r and h_p indicate the thickness at which R and PL become extreme for the first time.

INDEX = 1 GAMMA = 0.8

θ(°) →	0.		15.		30.		45.		60.	
H(cm)	Mag	Phase	Mag	Phase	Mag	Phase	Mag	Phase	Mag	Phase
0.0	0.847	0.002	0.842	0.002	0.825	0.003	0.790	0.004	0.716	0.008
0.5	0.841	-0.011	0.836	-0.011	0.819	-0.009	0.785	-0.005	0.716	0.008
1.0	0.829	-0.025	0.824	-0.024	0.807	-0.021	0.775	-0.013	0.716	0.008
1.5	0.808	-0.039	0.803	-0.038	0.787	-0.033	0.758	-0.021	0.716	0.008
2.0	0.777	-0.053	0.772	-0.051	0.759	-0.043	0.737	-0.027	0.716	0.008
2.5	0.732	-0.066	0.729	-0.062	0.719	-0.052	0.709	h_p $\underline{-0.030}$	0.716	0.008
3.0	0.671	h_p $\underline{-0.074}$	0.670	h_p $\underline{-0.069}$	0.670	h_p $\underline{-0.055}$	0.679	0.029	0.716	0.008
3.5	0.597	-0.069	0.601	-0.064	0.616	-0.047	0.649	-0.022	0.716	0.008
4.0	0.529	-0.040	0.539	-0.036	0.570	-0.024	0.628	-0.008	0.716	0.008
4.5	h_r $\underline{0.503}$	0.014	h_r $\underline{0.516}$	0.014	h_r $\underline{0.555}$	0.012	h_r $\underline{0.621}$	0.010	0.716	0.008
5.0	0.541	0.065	0.550	0.060	0.578	0.047	0.631	0.028	0.716	0.008
5.5	0.613	0.088	0.616	0.082	0.627	0.066	0.655	0.040	0.716	0.008
6.0	0.686	0.089	0.684	0.084	0.681	0.071	0.685	0.046	0.716	0.008
6.5	0.743	0.079	0.739	0.076	0.729	0.066	0.715	0.046	0.716	0.008
7.0	0.785	0.066	0.780	0.064	0.766	0.057	0.742	0.042	0.716	0.008
7.5	0.813	0.052	0.808	0.051	0.792	0.046	0.762	0.036	0.716	0.008
8.0	0.832	0.038	0.827	0.038	0.810	0.035	0.777	0.028	0.716	0.008
8.5	0.843	0.025	0.838	0.024	0.821	0.023	0.787	0.019	0.716	0.008
9.0	0.847	0.011	0.842	0.011	0.826	0.011	0.790	0.010	0.716	0.008
9.5	0.846	-0.002	0.841	-0.002	0.824	-0.001	0.789	0.001	0.716	0.008
10.0	0.838	-0.016	0.833	-0.015	0.816	-0.013	0.783	-0.007	0.716	0.008

Table 1 is prepared to list R and (PL/π) to 3 decimals at 0.8, 1.0, 1.2 and 5 GHz with incident angle θ from 0 to 60° by 15° step. For oil (index 1) and fresh water (index 2), a part of the printout is shown here, and the whole table is available upon request.

Figure 2 shows the curves of R and PL versus the parameter β_m denoted as β for the oil m=1 and the fresh water m=2 at the incident angle 0° and 45°. It is seen that R changes with β more strongly for oil than for water, whereas PL changes with β the other way. Also changes of R with β are less conspicuous for increasing angle. This figure suggests that to determine the thickness of the film with decrease of R is more feasible for oil than for fresh water, whereas PL change may be useful to determine the freshwater cover.

It is more practical to plot R and PL versus h in cm. Table 2 is prepared to list R and (PL/π) for h=0 to 10 cm at 0.1, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, and 5 GHz for oil and fresh water for index 1 and 2, respectively with incident angle ranging 0 to 60° of 15° step. A part of the table is shown with the whole available upon request. Some examples are shown in Fig. 3 as R and PL versus thickness h in cm.

Figure 3 and Table 2 indicate that R and PL change little against change of thickness h up to 10 cm for the frequency less than 0.4 GH$_2$ or lower but at 5 GHz both change rapidly with slight change in h. Also R and PL decrease or increase, respectively as θ increases for small values of h but as h increases this does not hold. On the other hand for the water at 0.1 GHz the R and PL show the decrease with the thickness h. Also the change of R and PL with h for different angle θ is more complicated for the water than for the oil.

Further, R and PL decrease with increasing incident angle up to 60°. For oil and fresh water, R decreases and PL increases with increasing thickness up to a certain thickness. This critical thickness is denoted as h_r and h_p for R and PL, respectively. Then both h_r and h_p depend on frequency and incident angle. In practical term these depths may be considered as a limit for using R or PL of reflected waves to determine the depth of oil or fresh water covering the sea surface. Fig. 3 and Table 2 indicate that both h_r and h_p are larger for fresh water than for oil at the same frequency range and that h_r is larger than h_p for the same frequency for oil but practically the same for the fresh water. However, values of h_r and h_p are almost independent on the inci-

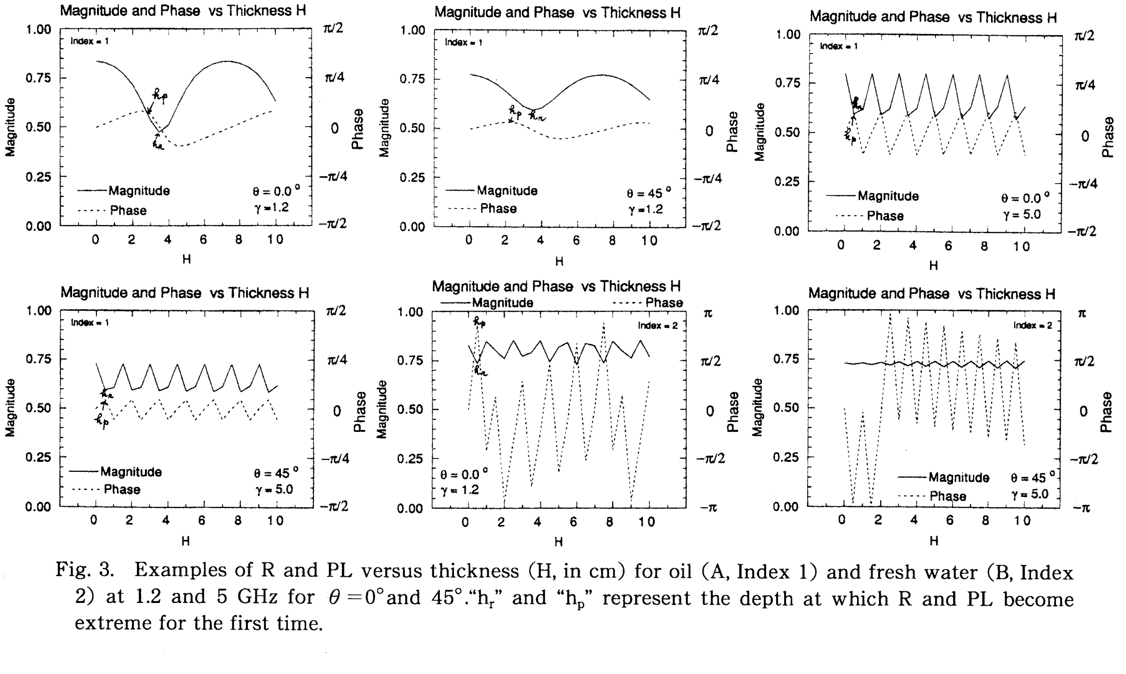

Fig. 3. Examples of R and PL versus thickness (H, in cm) for oil (A, Index 1) and fresh water (B, Index 2) at 1.2 and 5 GHz for θ=0° and 45°. "h_r" and "h_p" represent the depth at which R and PL become extreme for the first time.

dent angle. Also both are less for fresh water than for oil at the same frequency and the same incident angle.

If h is intended to measure with change of R and PL of reflected waves, oil is easier to do so than water, because the range of h which makes this possible is larger for oil than fresh water. This is in accordance with the intuitive judgment, since oil is more different in electro-magnetic properties from sea water than fresh water is, with oil being different both in conductivity and dielectric constant, whereas freshwater having the same dielectric constant as the sea water.

5. Very high and low frequencies

As discussed in Section 3, media in which EMW propagates behave as conductor or insulator according to frequencies of incident EMW. However, reflection of EMW at the sea surface covered by oil or fresh water depends on the behavior of the sea water in relation to EMW frequency, since oil and fresh water behave as insulator at microwave frequencies near or higher than 1 GHz because conductivity of these media is several orders lower than that of sea water. This can be seen from equations (21) and (22) which do not contain terms depending on δ_1. These two equations do contain u and v that depend on δ_2 through equation (19) or (23e).

However, conductivity of oil or fresh water may reach that of sea water when mixture between the two progresses. In that case the approximations (21) and (22) are not valid and results of Appendix I should be used.

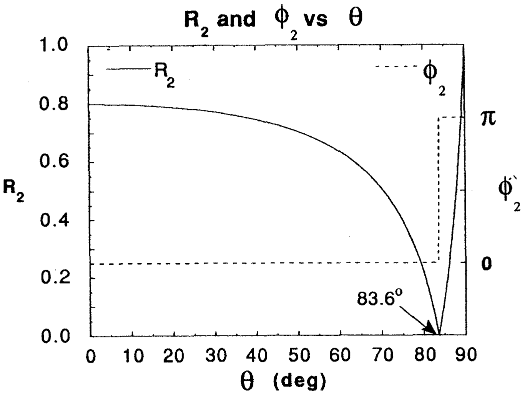

Fig. 4. R and PL versus incident angle θ for fresh water from equation (25).

Equation (23e) indicates that

$$u \approx 1, \quad v \approx 0 \tag{24}$$

for $\Gamma \geqq 15$ within an error of 6%. Then (21) and (22) become simple and for freshwater (m=2), from equation (20),

$$R_2 e^{-i\phi_2} = (\cos\theta - p)(\cos\theta + p)^{-1}. \tag{25}$$

Therefore PL is zero for θ less than 83.6° and is π for θ larger than this angle. Fig. 4 shows curves of R_2 and ϕ_2 versus θ. The values of R_2 and ϕ_2 do not depend on h and indicate reflection simply at the sea water surface without the upper layer. Therefore microwaves of frequency higher than 15 GHz are not useful to monitor presence of fresh water over the sea surface.

On the other hand, for oil the approximations (24), (23a) and (23b) lead to

$$R_1 e^{-i\phi_1} = \{(\cos\theta - p)\cos\beta - i(0.5 - 2p\cos\theta)\sin\beta\} \times \{(\cos\theta + p)\cos\beta + i(0.5 + 2p\cos\theta)\sin\beta\}^{-1}, \tag{26}$$

where $\beta = \beta_1$ of (23c). When (26) is averaged over β from $n\pi$ to $(n+1)\pi$, the r.h.s. of (26) becomes simply by approximation of (23a) and (23b) (See Appendix II for derivation),

$$\overline{R_1} e^{-i\overline{\varphi_1}} = \{(1+2p)\cos\theta - p - 0.5\} \times \{(1+2p)\cos\theta + p + 0.5\}^{-1} \tag{27a}$$

or returning to dielectric constants of oil and sea water, ε_1 and ε_2

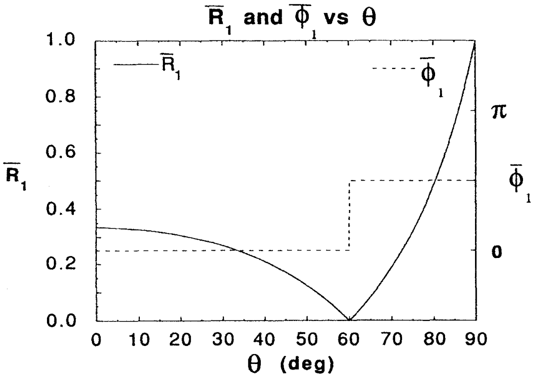

Fig. 5. R and PL averaged over beta versus incident angle θ for oil from equation (27 a) or (27b). (The dotted lineshould jump to π at $\theta = 60°$)

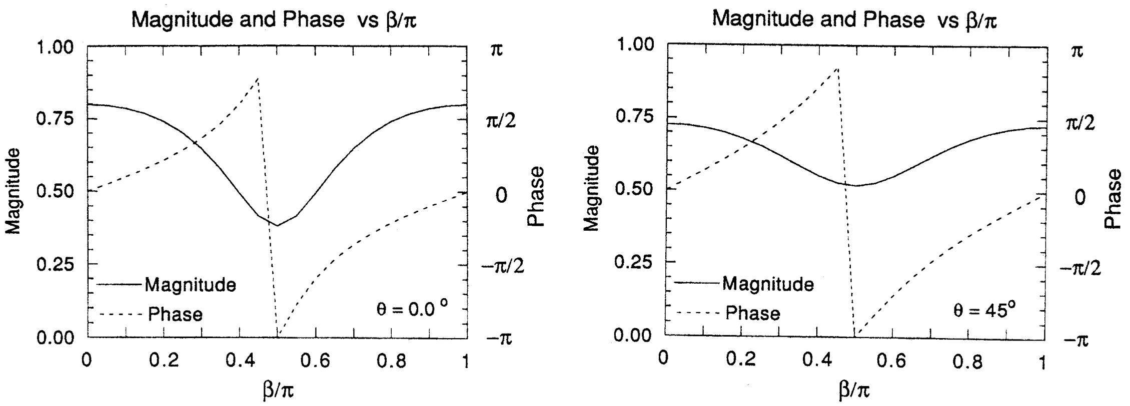

Fig. 6. R and PL versus beta for frequency range higher than 15 GHz for $\theta = 0°$ and 45°. (Compare with Fig. 2)

$$\overline{R}_1 e^{-i\overline{\varphi}_1} = [\{1+(\varepsilon_1/\varepsilon_2)^{\frac{1}{2}}\}\cos\theta - \{(\varepsilon/\varepsilon_1)^{\frac{1}{2}}(\varepsilon/\varepsilon_2)^{\frac{1}{2}}\}]\times$$
$$[(1+(\varepsilon_1/\varepsilon_2)^{\frac{1}{2}}\}\cos\theta + \{(\varepsilon/\varepsilon_1)^{\frac{1}{2}}+(\varepsilon/\varepsilon_2)^{\frac{1}{2}}\}]^{-1}, \quad (27b)$$

where ε is dielectric constant of vacuum.

Curves of $\overline{R}_1$ and $\overline{\varphi}_1$ versus θ are plotted in Fig. 5 based on equation (27a). Both parameters depend only on θ, since they are integrated over h. The $\overline{R}_1$ decreases with increasing θ as seen in Fig. 3 for lower frequencies but for the same value of h. On the other hand, $\overline{\varphi}_1$ is zero for $\theta \approx 60°$ and then jumps to π. In terms of dielectric constant of oil, equation (27b) leads to

$$\overline{\varphi_1} = 0 \quad \text{for } 0 \leq \theta < \text{Arctan}\,(\varepsilon/\varepsilon_1)^{\frac{1}{2}}$$
$$= \pi \text{ for Arctan}\,(\varepsilon/\varepsilon_1)^{\frac{1}{2}} \leq \theta < \pi/2. \quad (27c)$$

The averaging over π on phase β of equation (26) is justified, because at 15 GHz about 5 mm change in h can produce change by π in β from equation (23c). This is based on an assumption that in the field thickness of the oil film is not uniform but its variance may reach 5 mm.

When wave frequencies higher than 20 GHz are used, identification of oil over the sea may be possible if its dielectric constant identifies the kind of oil. This is to apply equation (27 b) to measured $\overline{R}_1$ that becomes useful if the oil thickness has a variance of a few mm. For scanning a narrow field with much less thickness variances, equation (26) may be used to determine thickness by measuring R_1. PL from equation (26) fluctuates rapidly with change of h as discussed before, thus it is doubtful that PL may be used to determine h. Fig. 6 shows R_1 and φ_1 versus β.

For EMW with frequencies less than 0.1 GHz, relations (21) and (22) become simplified again, since from equation (19)

$$u \approx v \approx \kappa\,(s_2\omega/\delta_2)^{\frac{1}{2}}. \quad (29)$$

Further sine and cosine functions of (21) and (22) can be approximated by

$$\sin\beta_m \approx \beta_m \qquad \cos\beta_m \approx 1, \quad (30)$$

However, for oil the low frequency EMW causes very little change in R and PL with increasing h up to 10 cm. Thus possibility of determining h with EMW of 0.1 GHz or lower frequencies is practically slight.

For the fresh water, 0.1 GHz wave may be utilized to determine h with reduction in R and increase in PL with increasing h. The very low frequency approximation for (21) and (22) is simple substitution of (27) and (28) into these equations, thus it is not presented explicitly here.

However, there is a possibility that both oil and fresh water may change their electromagnetic properties, particularly their conductivity by mixing with underlying sea water as time progresses. Therefore, no more valid is the approximation $\omega^{-1}\delta_1 \ll 1$ that leads to equations (21) and (22). In order to compute in such cases, expression of b_0/a_0 without approximations of (17) and (18) but with (15) and (16) is given in Appendix I. This may be used to give corrections to Fig. 2 for determing R and PL, if, σ_1

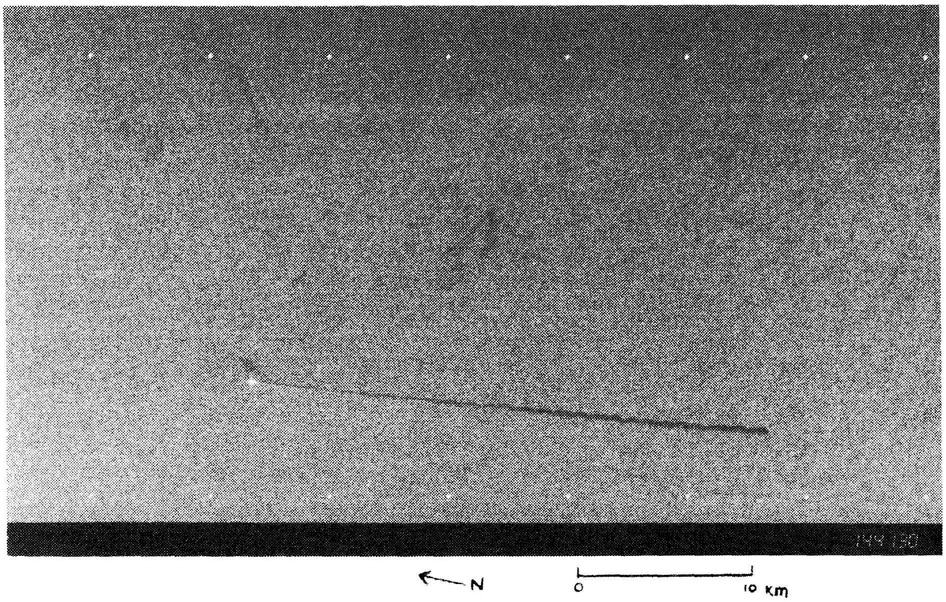

Fig. 7. Wake of a ship that was supposedly discharging oil or oil contaminated liquid. This ship was about 500 km from the coast and heading for Delaware Bay (p. 116 in Fu and Holt, 1982). (The glossy print is courtesy of Dr. Fu at JPL.)

and ε_1 of oil or fresh water is known by aging or mixing with the sea water. Inversely, this relation can be used to estimate of aging of oil or mixing rate of the fresh water with the sea water by measuring R and PL of reflected EMW at the sea surface.

6. Applications and concluding remarks

The R and PL of EMW reflected at the sea surface covered with oil or fresh water show significant departure from those of the uncontaminated sea surface. Some SAR images taken with SEASAT-satellite in 1978 showed the oil covered sea surface with a black patch as indicated in Fig. 7 that is duplicated from report by Fu and Holt (1982). This has been interpreted as R being diminished due to suppression of capillary waves and ripples by oil film. However, as indicated in Fig. 3, R is reduced by 15 to 18 % from the clean sea surface with presence of oil film a few mm thick for EMW of 1.4 GHz. Therefore reduction of R of plane EMW due to a thin film should be considered to account for blackness of oil covered sea.

On the other hand R and PL decrease or increase almost linearly with thickness of oil or fresh water up to its critical value of a few centimeters depending on the frequency of incident EMW. The critical thickness is larger for water than oil and becomes smaller at higher frequencies in GHz range. Therefore, SAR or SLAR may be used to determine thickness of oil or fresh water over the sea by determing of R or PL of reflected EMW.

Since present SAR and SLAR use the frequency range that is adequate for determining R or PL of expected thickness, they can measure thickness of oil in case of a major oil spill or that of freshwater and precipitation on the Tropical Pacific Ocean to forewarn El Niño processes.

With about 100 GHz or higher frequencies, if variance of oil thickness is an order of 0.1 mm or more over a relatively wide area, the aging of spilled oil at the sea surface may be determined through change of its dielectric constant by measuring R as indicated by equation (27b). At the currently available frequency range of 1 to 20 GHz, if R and PL of reflected waves can be determined accurately, not only thickness h of oil or fresh water covering the sea surface may be measured but also its aging or mixture with the sea water could be determined. In the latter case, approximate equations (21) and (22) are no more valid but relations presented in Appendix I should be applied.

Acknowledgement

Numerical computations and computer graphics were provided by Matthew Howard. Publication of this paper is partially suported by U. S. WOCE program (headed by W. Nowlin, Dept. of Oceanography, Texas A & M University).

Appendix I

Substitution of (12), (15) and (16) into (8 b) and (8c) lead to

$$b_0/a_0 = P/Q \tag{A-1}$$

$$P = (\cos\theta - P_1)\cos\beta_m - i(P_2 - \cos\theta P_3)\sin\beta_m \tag{A-2}$$

$$Q = (\cos\theta + P_1)\cos\beta_m + i(P_2 + \cos\theta P_3)\sin\beta_m \tag{A-3}$$

where

$$P_1 = s_2^{-\frac{1}{2}}(1 - s_2^{-1}\sin^2\theta - \delta_2\omega^{-1}i)^{\frac{1}{2}}(1 - i\delta_2\omega^{-1})^{-1} \tag{A-4}$$

$$P_2 = (s_{1,m})^{-\frac{1}{2}}(1 - (s_1, m)^{-1}\sin^2\theta - s_{1,m}\omega^{-1}i)^{\frac{1}{2}}(1 - i\delta_{1,m}\omega^{-1})^{-1} \tag{A-5}$$

$$P_3 = P_2/P_1. \tag{A-6}$$

$$\beta_m = s_{1,m}(\omega/c)\{1 - (s_1 m)^{-1}\sin^2\theta - \delta_{1,m}\omega^{-1}i\}^{\frac{1}{2}} \tag{A-7}$$

In equations (A-4), (A-5) and (A-7), m=1 and 2 represent oil and fresh water respectively. Note m-suffix is added to s_1 and δ_1, the upper layer parameters corresponding to s_2 and δ_2 of the sea water. Therefore, $s_{1,1}$ corresponds to $s_1$in (23b) and (23c) and $s_{1,2} = s_2$. When aging of oil and mixing of fresh water with sea water progress, range of $s_{1,1}$ changes from s_1 to s_2. Further those of δ_1 from 10^{-7} to 0.9 GHz and of $\delta_{1,2}$ from 10^{-4} to 0.9 GHz, respectively, though electromagnetic properties of oil $s_{1,1}$ and σ_1, are less known after aging than those of rain water after mixing with sea water.

Appendix II

For high frequency, (23c) and (23d) indicate β_1 and β_2 become large for small change in h. For example, at $\Gamma = 20$, β_1 and β_2 change by one cycle (2π) for variances of h of 1.3 cm and 0.17 cm respectively. When SAR or SLAR scans the sea surface, it covers a sufficient swath area to include variances of h of this orders of magnitude. Then the reflected wave consists of many component waves reflected at oil or fresh water of various thicknesses, variances of which make those of β_1 or β_2 2π or much larger. This is different from reflection or scattering of EMW from rough surfaces treated by a monograph of Beckmann and Spizzichino (1963), because all reflected wave components in this case are reflected with the same angle. The irregularities in h are random but continuous.

Thus R and PL of the effective reflected waves are expressed by averaging components waves with β_m over interval π as

$$R_m e^{-i\varphi m} = \frac{1}{\pi}\int_0^{\pi}(M_m/N_m)\,d\beta_m, \tag{B1}$$

where M_m and N_m are given by (21) and (22), since M_m and N_m are repeated over π to 2π.

Hereafter, suffix m is dropped. Then (B1) becomes

$$Re^{-i\varphi} = \frac{1}{\pi}\int_0^{\pi}(a\cos\beta - i\,b\sin\beta) \times (A\cos\beta + i\,B\sin\beta)^{-1}d\beta \tag{B2}$$

$$Re^{-i\varphi} = \{\pi(A^2 - B^2)\}^{-1}[a\{A\beta + iB\log\times(iB\sin\beta + A\cos\beta)|_0^{\pi}\} - ib\{iB\beta - A\log(iB\sin\beta + A\cos\beta)|_0^{\pi}\}] \tag{B3}$$

$$a = \cos\theta - p, \quad b = 0.5 - 2p\cos\theta \tag{B4}$$

$$A = \cos\theta + p, \quad B = 0.5 + 2p\cos\theta \tag{B5}$$

and in the integrated expression the logarithm is generalized to a complex argument by use of analytic continuation from formulas listed for a real argument (Moriguchi *et al.*, 1957, p. 191-192). In integration from $\beta = 0$ to π, the argument of log-terms in the complex plane changes from (A, 0) at $\beta = 0$ through (0, iB) to (-A, 0) at $\beta = \pi$. Therefore, the real and imaginary part of log-term is given by

$$\log(iB\sin\beta + A\cos\beta)\Big|_0^{\pi} = \log|-A| - \log|A| + i\pi = i\pi. \tag{B6}$$

Thus (B-3) becomes simply

$$Re^{-i\varphi} = (a - b)(A + B)^{-1} = (\cos\theta - 0.5)(\cos\theta + 0.5)^{-1}. \tag{B7}$$

References

Beckmann, P. and A. Spizzichino (1963): The Scattering of Electromagnetic Waves from Rough Surfaces. Pergamon Press. 503pp.

Elachi, C. (1988): Spaceborne Radar Remote Sensing: Application and Techniques. IEEE Press. 255p.

Fu, L-L. and B. Holt (1982): Seasat Views Oceans and Sea Ice with Synthetic-Aperture Radar. NASA, JPL Publ. 81-150. 200pp.

Gill, A.E. (1983): An estimation of sea-level and surface-current anomalies during the 1972 El Niño and consequent thermal effects. J. Phys. Oceanog., **13**, 586-606.

Ichiye, T. and K. Muneyama (1989): Upper layer low salinity pools in the western Tropical Pacific Ocean, (Submitted to La mer).

Kay, G.W. and T.H. Laby (1988): Tables of Physical and Chemical Constants. p. 109, Longman Co.

Landolt and Börnstein (1960) and (1986): Numerical Data and Functional Relationships in Science and Technology. New Series, part 7, p. 17, and Vol. 3 part 9, p. 237. Springer Verlag, Berlin.

Liebermann, L. N. (1962): Other electromagnetic radiation. The Sea. Vol. 1 (edited by M.N. Hill). John Wiley & Sons, N.Y. p. 469-475.

Moriguchi, S., K. Udagawa, and N. Ichimatsu, (1956): Mathematical Formulas I. Iwanamishoten. 318pp.

Phillips, O.M. (1987): Remote sensing of the sea surface. Ann. Rev. Fluid Mech. 20, 89-109.

Slater, T. C. (1942): Microwave Transmission. Dover. 309pp.

Stratton, J. A. (1941): Electromagnetic Theory. McGraw-Hill, NY. 615pp.

Trebits, R.N.(1987): Synthetic Aperture Radar. Principles of Modern Radar (edited by T.L. Eaves and E. K. Reedy), Van Nostrand Rheinhold, N.Y. p.502-537.

Wait, J. R. (1962): Electric Magnetic Waves in Stratified Media. Pergamon Press. 372pp.

海面での電磁波の反射

市　栄　誉

要旨 ：平面電磁波の海面での反射をマックスウエルの理論から取扱った。原油又は淡水が一様の厚さで無限に深い海を覆っていると仮定する。原油又は淡水の電媒常数と電気伝導率では反射波の反射率は上層の厚さによって殆んど一次的に減じ、その位相のおくれは逆に増える。従って原油を流出した時に油の厚さあるいはエル・ニーニヨの前に熱帯太平洋の降水による表面の水の厚さは約1ギガヘルツのSARまたはSLARで測定されうる。また、これらの厚さがほかの方法で知られている場合には反射率か位相差を正確に測ることにより油の古さや雨水の海水との混合など油又は水の電導度と電媒常数から推定されうる。20ギガヘルツ以上の周波数の電磁波は淡水の存在を知るには不適当だが、油の老化を電媒常数から推定するのに使える。これは電磁波はもし原油の厚さに数ミリメートル程度の偏差があれば異なった反射率をもつが、同じ反射角で反射し，そのため平均反射率は入射角と油の電媒常数だけによるからである。